Comments on "Reservoir lithology stochastic simulation based on Markov random fields"

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Liang et al. (2014) stated that they proposed "Markov random fields" (MRF) and a Gibbs distribution to simulate three-dimensional (3D) reservoir lithology and that they considered maximum likelihood approaches of model parameters on well data and training image. Browsing the paper, one can see that they indeed talked about MRF, introduced some knowledge about MRF, and also talked a lot about maximum likelihood estimation.

However, carefully reading the whole paper, we find a fact - the only thing they did was implementing a 3D unconditional coupled Markov chain (CMC) model, which was presented as Equation (8) in their paper. This model is not relevant with most of other contents they introduced. They stated that they rewrote the conditional probability distribution of the 3D spatial Markov chain at any unknown location as Equation (8):

$$P\left(x_{i,j,k}=l \middle| x_{i-1,j,k}=l_1, x_{i,j-1,k}=l_2, x_{i,j,k-1}=l_3\right) = p^x_{l_1 l} p^y_{l_2 l} p^z_{l_3 l} / \sum_l p^x_{l_1 l} p^y_{l_2 l} p^z_{l_3 l} \quad (8)$$

with a citation to Li (2007). Apparently they implied that Equation (8) was a Markov chain random field (MCRF) model proposed by Li (2007), in which a single-chain-based 3D conditional Markov chain model (i.e., a 3D MCRF model) for nearest data in six cardinal directions was also provided as a specific case of the MCRF model. However, checking the transition probability terms, it is not difficult to find that Equation (8) is clearly the unconditional CMC model in 3D, extended from the 2D unconditional CMC model proposed by Elfeki (1996). Park et al. (2005) extended the 2D conditional CMC model of Elfeki and Dekking (2001) into a 3D conditional CMC model, but did not properly test the 3D model. Equation (8) in Liang et al. (2014) is actually an unconditional version of the 3D conditional CMC model, which comprises three 1D Markov chains.

When they implemented their "MRF" model, they exactly implemented Equation (8), as they stated in *Step 3* of their simulation procedure (see Liang et al. 2014, p. 3615). Their simulation procedure is non-iterative, but they did not tell what simulation path they used. Elfeki and Dekking (2001) extended the unconditional CMC model into a conditional CMC model, that is, they made the coupled Markov chains conditional to borehole data ahead. However, Liang et al. (2014) just implemented an unconditional version (used only three immediately adjacent grid cells). They showed two sets of simulated realizations. One set was generated by Equation (8) and presented in their Figure 3, as they stated. They stated that this set of simulated realizations was the results by MRF. The other set of simulated realizations was presented in their Figure 2, and they stated that it was produced by a "Markov chain model". But they did not tell what Markov chain (MC) model they used. According to the simulated results in the two figures, MRF generated quite good results, while MC strongly underestimated small classes.

Here questions with this paper are: (1) how could they get simulated realizations without small class underestimation by their MRF while they implemented an unconditional CMC model? (2) what is the 3D MC model they used to generate the simulated realizations with strong small class underestimation? (3) what is the simulation algorithm they used for implementing the unconditional CMC model? (4) why did they regard the unconditional CMC model to be MRF while implying it was a MCRF model? It seemed that they were completely confused on the CMC model, MCRF, MRF and Markov chains. Consequently they wrote their paper very vaguely about what they did and their results.

Because the CMC model underestimates small classes, Li (2007) proposed the MCRF model to solve the problem. The small class underestimation tendency of the CMC model was displayed by Elfeki and

Dekking (2001), although they did not specifically discuss it. While Liang et al. (2014) implemented the unconditional CMC model in 3D, they did not mention the name of the "coupled Markov chain" model, although they did cite the paper of Elfeki and Dekking (2001) in their paper. Maybe they wrote Equation (8) mistakenly because they could not understand it, or maybe they really thought that the unconditional CMC model was a MRF model while it was actually built on three 1D Markov chains. However, the paper of Liang et al. (2014) indeed conveyed misleading information by mixing the basic knowledge of MRF, the equation of the CMC model, the unexplained MC model, and the citation of Li (2007) for the MCRF model together in their paper.

References

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